UNITED STATES NAVAL OBSERVATORY

CIRCULAR NO. 30

U. S. Naval Observatory, Washington 25, D. C. May 11, 1951

SUNSPOT AREAS AND NUMBERS AS INDICES OF SOLAR ACTIVITY

By Wm. Markowitz

INTRODUCTION

This circular has been prepared in answer to requests for information regarding sunspot measurements made at the U. S. Naval Observatory, and the relation of these results to those obtained at other observatories. There are included basic definitions concerning sunspot areas and relative sunspot numbers, graphs of these quantities, and brief descriptions of the methods used by some of the principal agencies which publish solar data.

It is known that observational circumstances affect the determination of sunspot numbers to a considerable extent, and that of areas to a lesser degree. In order to obtain some quantitative idea as to how the various factors do influence the results it was decided to study some of these points in more or less detail. Results from plates taken at Washington and at Mount Wilson were compared in order to study effects of instrumental power and seeing, and the relative spot numbers determined at Washington and at Zurich were compared in order to study changes in scale. The relation between sunspot numbers and areas was also investigated.

DEFINITIONS

The observed area of a spot is the projection of the true area, A, on the plane perpendicular to the line of sight. The true area is obtained by correcting for foreshortening, and is expressed in millionths of the sun's hemisphere. The projected area, when given, is expressed in millionths of the apparent disk.

The number of groups of spots, each group containing one or more spots, is denoted by g, and the total number of spots, counting each umbra as a spot, is denoted by s. The relative sunspot number as defined by Rudolph Wolf of Zurich is

$$r=10g+s.$$

In order to reduce various sets of observations to the same system a reduction factor, k, was introduced. The reduced relative number is

$$R=kr.$$

The value of k, which is empirically determined, depends upon the equipment used and the personal equation of the observer. A small r is used in this circular to indicate the initially observed values and a capital R to indicate reduced values. It will be noted that sunspot areas and relative sunspot numbers are independent quantities, except that both become zero together.

Solar data are usually smoothed initially by taking monthly means. The solar rotation period is so nearly a month that variations arising from the sun's rotation are largely smoothed out. Additional smoothing is sometimes done by taking 12-month overlapping means. The smoothed means published by Zurich are obtained by taking the mean of two such consecutive 12-month means. The Zurich mean may also be obtained by taking the mean of 13 consecutive months, with half weight being given to the first and last months. The mean epochs of both the monthly and Zurich means fall on the middle of a month.

OBSERVATIONS AND PUBLICATIONS

(a) *U. S. Naval Observatory.* Photographs are taken daily with a photoheliograph of 40 feet focal length, using blue-sensitive plates. The heliographic positions of the spots and their areas are measured with the aid of a set of transparent grids. Both umbra and penumbra are included in the areas given. A description of the observing methods, and the results from 1917 to December 1927, are given in *Publications of the U. S. Naval Observatory*, Vol. XII, 2nd series. The results up to January 1948 were published in the *Monthly Weather Review*, issued by the U. S. Weather Bureau, and since then have been published in this series of circulars. An extended summary for each year, beginning with 1950, is to be issued.

Results for plates taken before 1917 have not been published, but the areas back to 1907 are given in Figure 1. The areas published for the years 1917 to 1937 were not on the present system (see *Monthly Weather Review*, February 1937, page 65), but all data given in this circular are based on the system of true areas defined on page 2. Counts of the number of spots and groups have been made since 1938.

For days on which photographs cannot be taken at Washington, plates furnished by Mount Wilson are used. The division of the spots into groups is based upon drawings supplied by Mount Wilson, which indicate the division into groups as made at Mount Wilson. Occasionally, groups containing but one spot cannot be found on the photographs although appearing on the charts, and vice versa.

(b) *Mount Wilson.* Direct photographs are made with the 12-inch lens of the 60-foot tower telescope, a set of which is sent to the U. S. Naval Observatory. A filter is used in conjunction with orthochromatic plates, the effective wavelength being about 5850A. The magnetic character of the spots is determined with the 75-foot spectrograph attached to the 150-foot tower telescope. Drawings are made at the focal plane, the solar image being about 17 inches in diameter. The magnetic data are given monthly in *Publications of the Astronomical Society of the Pacific;* descriptions of the methods used are given in volume 59, p. 36, 1947, and in the *Astrophysical Journal*, 49, 153, 1919; 62, 270, 1925.

(c) *Royal Observatory, Greenwich.* Photographs are made at Greenwich, supplemented by others taken at the Cape of Good Hope and at Kodaikanal, India. The positions and areas of sunspots, umbrae, penumbrae, and faculae, for each day are given in the annual volumes, *Greenwich Photoheliographic Results.* The results are given in considerable detail, but only those up to 1938 have been published at this date. Summaries for later years are given annually in the *Monthly Notices of the Royal Astronomical Society.*

(d) *Eidgenossischen Sternwarte, Zurich.* Visual observations are made of the number of spots and groups, using a 3-inch refractor with a power of 64. These observations are supplemented by those of cooperating stations when none can be made at Zurich. Provisional values of the relative sunspot numbers, Rz, are published in a number of places, such as the *Journal of Geophysical Research* (formerly *Journal of Terrestrial Magnetism and Atmospheric Electricity*), *Ionospheric Data* issued by the National Bureau of Standards, and *Popular Astronomy.* The final numbers as well as other solar results are published in the *Astronomische Mitteilungen der Eid. Stern. Zurich (A.M.).* Summaries of the Zurich numbers back to 1749 have been published in *A.M.* No. 145, 1945, *Terrestrial Magnetism* 44, 1939, and 53, 241, 1948. It should be noted that the Zurich numbers, which are now listed as " Beobachtete", are not those obtained directly from observation. A reduction factor of about $k=0.6$ has been used for some time in order to reduce the present results to the same system as those of earlier observations.

The Zurich Observatory also publishes the *Quarterly Bulletin on Solar Activity*, under the auspices of the International Astronomical Union, which gives sunspot numbers and other data concerning solar activity.

(e) *Central Radio Propagation Laboratory of the National Bureau of Standards.* A relative sunspot number, R_a, (American number) is computed daily from the results of about 40 cooperating stations here and abroad. Most of the measurements are made with small visual telescopes. The computational methods used are described in *Publications of the Astronomical Society of the Pacific*, **61**, 13, 1949. The results of the Zurich, Mount Wilson, and U. S. Naval Observatories are included, but with small weights relative to the total. The relative numbers R_a and R_z are published monthly in *Ionospheric Data*, and also as a separate table.

GRAPHS OF SUNSPOT AREAS AND NUMBERS

Figure 1 shows the mean daily area for each month from January 1907 to December 1950 as obtained at the U. S. Naval Observatory. In order to show some detail at the minima the vertical scale has been made non-linear, the ordinates being proportional to the square roots of the areas.

Figure 2 shows the annual means of the daily values of the smoothed Zurich sunspot numbers from 1750 to 1949.

Figure 3 compares the monthly means of r_w versus R_z, that is, observed Washington numbers versus reduced Zurich numbers for January 1938 to December 1950.

Figure 4 compares the Washington areas with the smoothed Zurich numbers from July 1907 to June 1950. The vertical scales, which are linear, are such that an area of 1000 has the same ordinate as a sunspot number of 60. The areas have been smoothed in the same way as the Zurich numbers. Attention is called to the high correlation between the quantities plotted.

INSTRUMENTAL POWER AND SEEING

The Naval Observatory results are based upon plates taken at Washington and at Mount Wilson. Those from Mount Wilson are taken with a larger telescope and, in general, under better seeing conditions. A comparison of the results from the two instruments was made for the period January 1948 to April 1949, one of high solar activity. The daily means are given in Table I.

TABLE I. INSTRUMENTAL EFFECTS.

	A	*g*	*s*	*r*
Mount Wilson	2080	10.3	79	182
Naval Observatory	2120	10.0	46	146
difference	− 40	+0.3	+33	+36

It is seen that the areas are practically alike, and also the group counts, but that the spot counts are quite different. The difference in the resulting sunspot numbers is about 23%. As about one-fifth of the plates used are Mount Wilson plates the average sunspot number derived at Washington is about 5% higher than if no Mount Wilson plates were used.

In order to see how serious the effects of variations in seeing can be the following example was selected. The results obtained by Mrs. L. T. Day for three successive days in April 1948 are given in Table II.

TABLE II. SEEING EFFECTS.

Date	Observatory	Plate quality	*A*	*g*	*s*	*r*
12	Naval Observatory	fair	3075	17	83	253
13	Mount Wilson	very good	3530	17	255	425
14	Mount Wilson	fair	3331	16	88	248

The large change in the spot count on the 13th is not reflected in similar changes in *A* and *g*, and the change is undoubtedly due to the excellent seeing on the 13th. It should be noted that this is an extreme case, occurring when numerous small spots were present on the sun.

To see what effect magnifying power has on the results, the plates for the 13th and 14th were remeasured by the writer using eye lenses of one and ten inches focal length. The total equivalent magnifications are 720 and 72 respectively. The high power is sufficient to show the smallest definite spot condensations, about 2″ in angular diameter, while the low power is similar to that used in making visual observations. The number of spots counted is given in Table III.

TABLE III. MAGNIFICATION AND SEEING.

Date	Plate quality	720 power	72 power	ratio
13	very good	420	170	2.5
14	fair	130	90	1.4

The use of high power increased the number of spots seen, but the ratio of increase was much greater when the seeing was good. It is evident that by using high power in making observations more spots can be seen, and greater day-to-day fluctuations in the spot count will occur than when using low power. In effect, therefore, the use of low power is a form of smoothing.

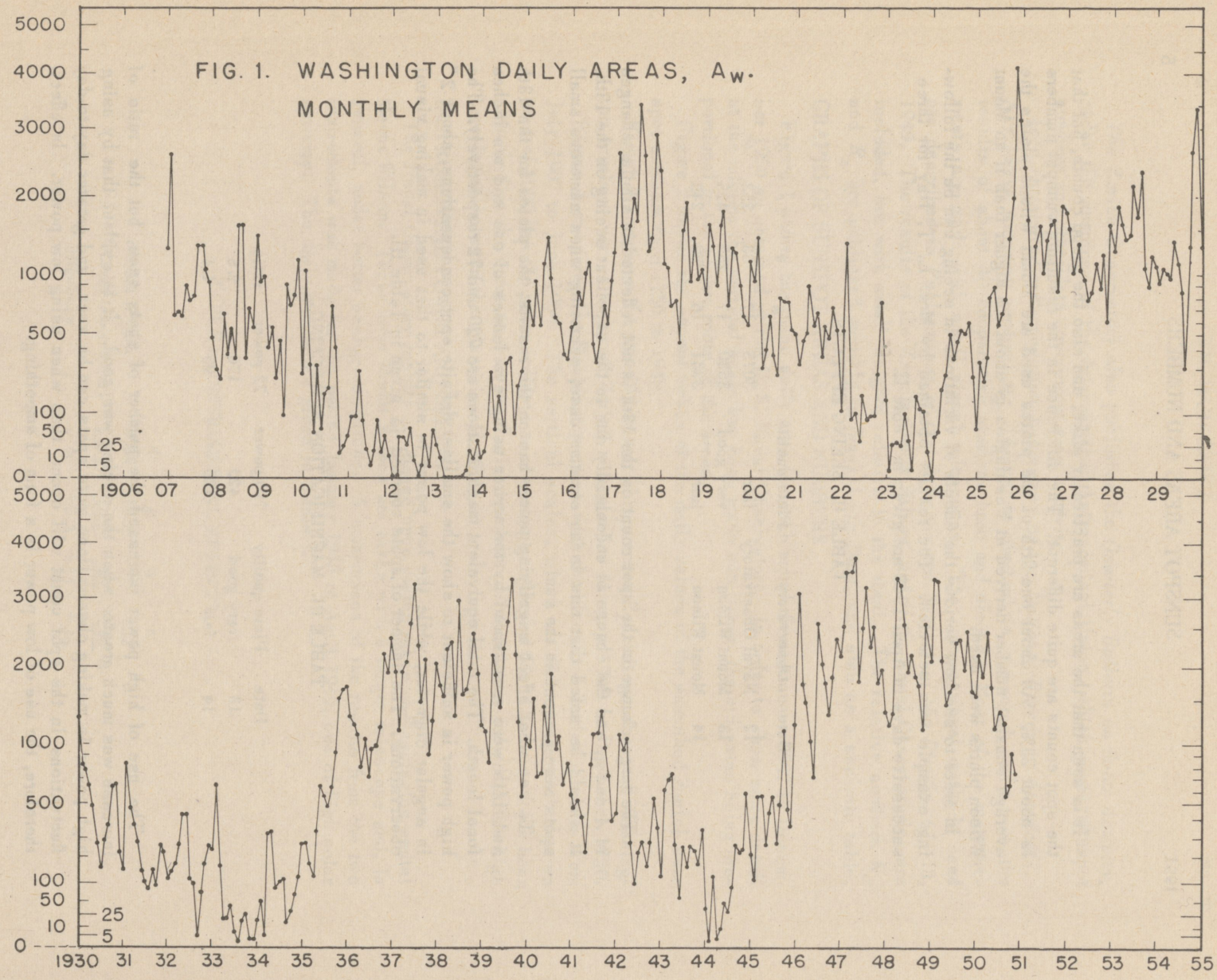
FIG. 1. WASHINGTON DAILY AREAS, Aw.
MONTHLY MEANS
5000
4000
3000
2000
1000
500
100
50
25
10
5
0
1906 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29
1930 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55

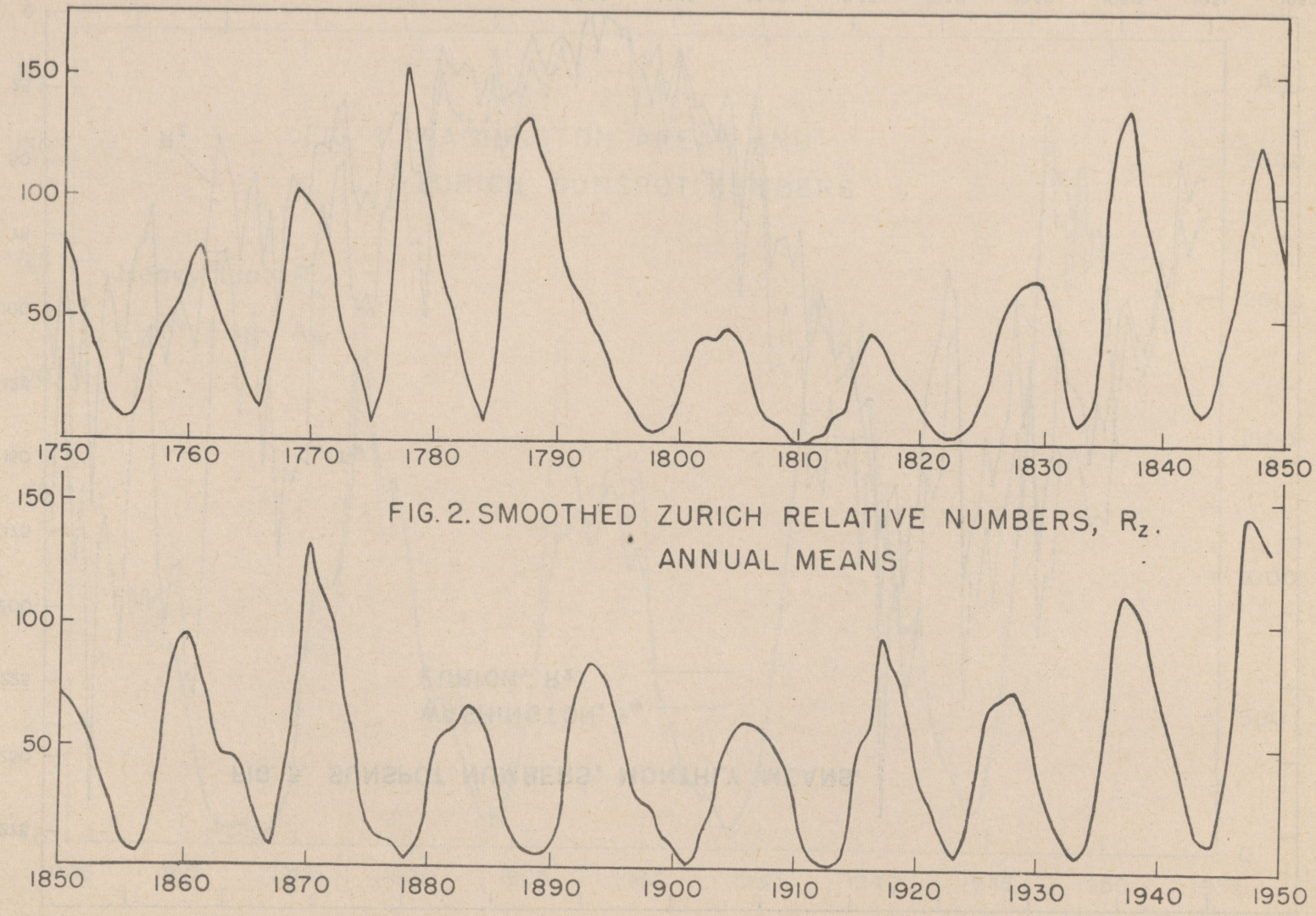

FIG. 2. SMOOTHED ZURICH RELATIVE NUMBERS, R_z.
ANNUAL MEANS

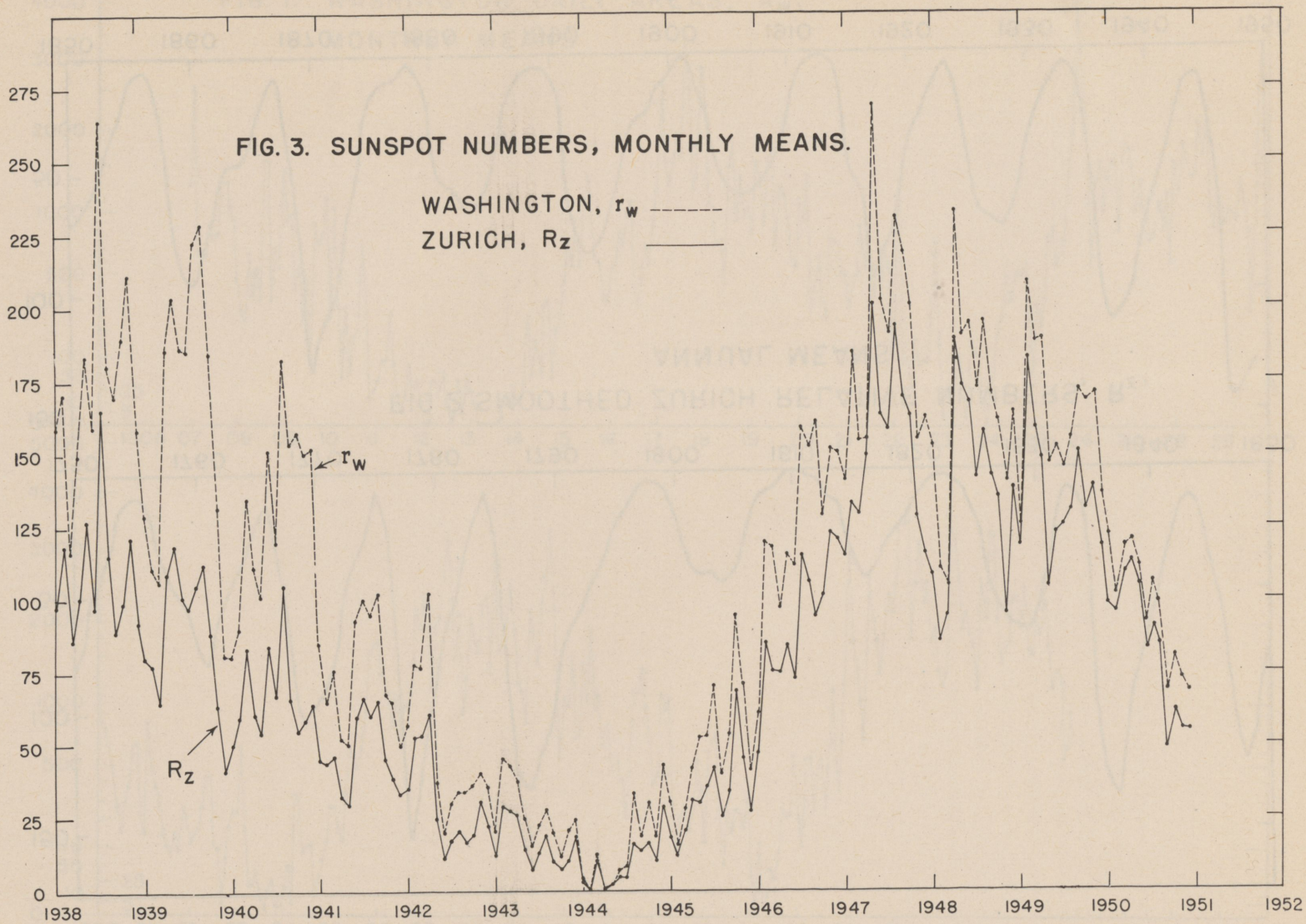
FIG. 3. SUNSPOT NUMBERS, MONTHLY MEANS.
WASHINGTON, r_w ------
ZURICH, R_z ——
r_w
R_z
275
250
225
200
175
150
125
100
75
50
25
0
1938
1939
1940
1941
1942
1943
1944
1945
1946
1947
1948
1949
1950
1951
1952

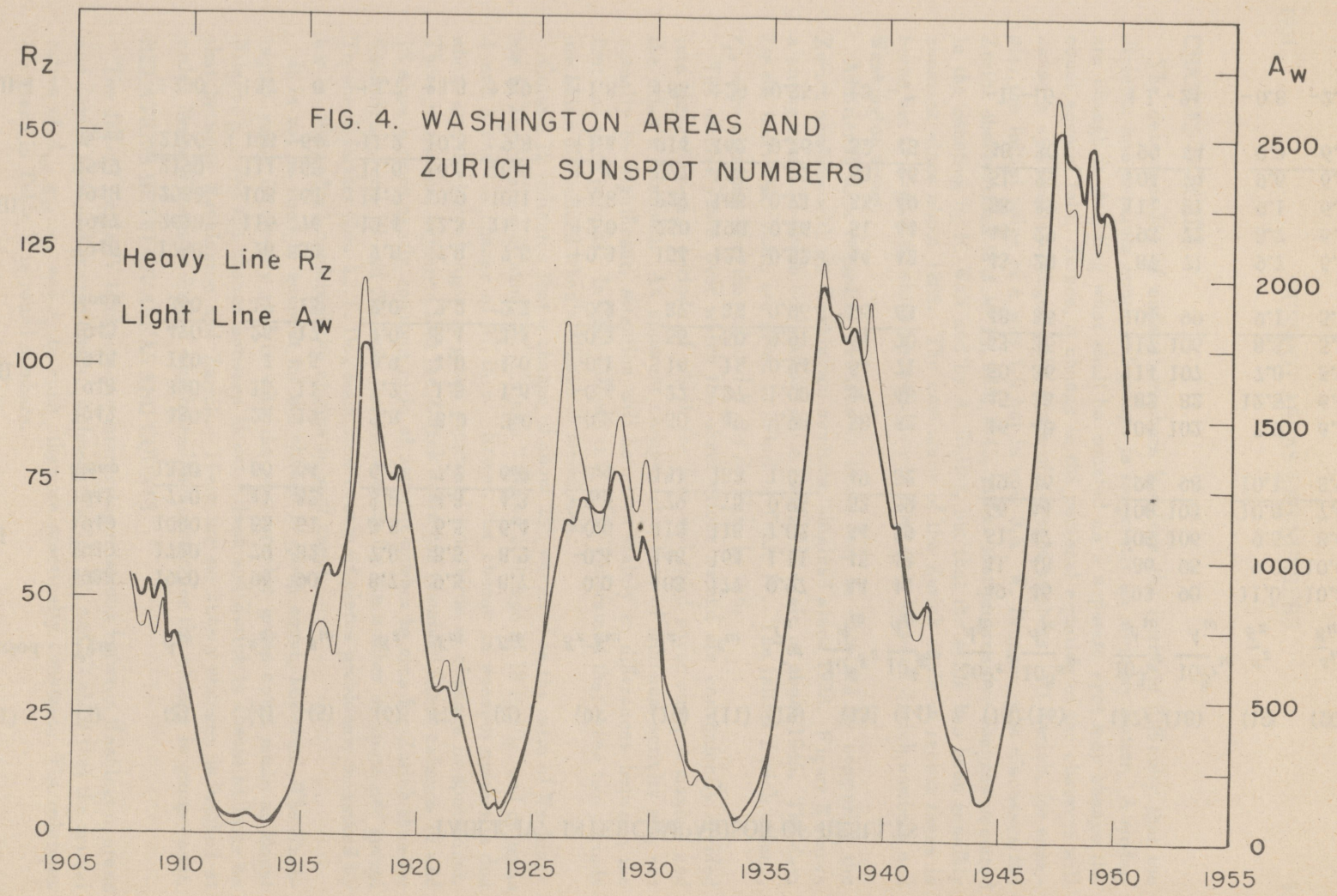
FIG. 4. WASHINGTON AREAS AND
ZURICH SUNSPOT NUMBERS
Heavy Line R_z
Light Line A_w
R_z
150
125
100
75
50
25
0
A_w
2500
2000
1500
1000
500
0
1905
1910
1915
1920
1925
1930
1935
1940
1945
1950
1955

TABLE IV. INTERCOMPARISON OF RESULTS

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Period	Year	A_w	s_z	s_w	g_z	g_m	g_w	g_z-g_w	r_z	r_w	$\frac{r_w}{r_z}$	$\frac{10^4 g_z}{A_w}$	$\frac{10^4 g_w}{A_w}$	$\frac{10^3 s_z}{A_w}$	$\frac{10^3 s_w}{A_w}$	$\frac{10^3 r_z}{A_w}$	$\frac{10^3 r_w}{A_w}$	$\frac{s_z}{g_z}$	$\frac{s_w}{g_w}$
	1938	1960	96	90	8.7	9.5	8.7	0.0	183	177	0.97	44	44	49	46	93	90	11.0	10.3
	1939	1720	70	82	7.8	8.5	8.2	-0.4	148	164	1.11	45	48	41	48	86	95	9.0	10.0
I	1940	1080	55	51	5.8	6.5	6.4	-0.6	113	115	1.02	54	59	51	47	105	106	9.5	8.0
	1941	730	41	32	3.8	4.3	4.3	-0.5	79	75	0.95	52	59	56	44	108	103	10.8	7.4
	Mean	1370	66	64	6.5	7.2	6.9	-0.4	131	133	1.01	49	52	49	46	98	98	10.1	8.9
	1942	480	22	19	2.8	3.0	3.0	-0.2	50	49	0.98	58	62	46	40	104	102	7.9	6.3
	1943	330	15	11	1.2	1.5	1.6	-0.4	27	27	1.00	36	48	45	33	82	82	12.5	6.9
II	1944	140	7	5	0.9	1.0	1.0	-0.1	16	15	0.94	64	71	50	36	114	107	7.8	5.0
	1945	470	25	17	3.0	3.4	3.3	-0.3	55	50	0.91	64	70	53	36	117	106	8.3	5.2
	Mean	360	17	13	2.0	2.2	2.2	-0.2	37	35	0.96	56	63	48	36	104	99	9.1	5.8
	1946	1780	76	52	7.8	7.8	7.5	+0.3	154	127	0.82	44	42	43	29	87	71	9.7	6.9
	1947	2620	116	76	13.4	12.3	11.4	+2.0	250	190	0.76	51	44	44	29	95	72	8.7	6.7
III	1948	2050	108	65	11.9	10.9	10.1	+1.8	227	166	0.73	58	49	53	32	111	81	9.1	6.4
	1949	2190	111	63	11.6	10.9	10.0	+1.6	227	163	0.72	53	46	51	29	104	74	9.6	6.3
	Mean	2160	103	64	11.2	10.5	9.8	+1.4	214	162	0.76	52	45	48	30	99	74	9.3	6.6
III-I		+790	+37	0	+4.7	+3.3	+2.9	+1.8	+83	+29	-0.25	+3	-7	-1	-16	+1	-24	-0.8	-2.3

CHANGES IN SCALE

The determination of sunspot numbers involves the intrinsic difficulty that the division into separate spots and groups is frequently an arbitrary matter. A moderate change in deciding what shall be counted as a separate spot may change the count several fold. Hence, a change in the personal equation of the observer will cause a systematic change in the scale of sunspot numbers. At Zurich, for example, the initial value of k was 1.0 but is now 0.6. W. Brunner, who discusses the Zurich observations in *A.M.* 15, 135, 1945, thinks that progressive changes occurred in the counting by R. Wolf and by A. Wolfer, but that the scale has been sensibly constant since about 1883. The constancy of the Zurich scale in recent times is borne out by Figure 4, which shows a nearly fixed ratio between Zurich numbers and Washington areas.

Additional data regarding changes in scale may be obtained by comparing the Zurich numbers with the Washington numbers, obtained since 1938. Figure 3 shows a strong month-to-month correlation, but a change in scale at about 1945. The data for the individual years are given in detail in Table IV. The initial Zurich values, r_z, were obtained from the reduced R_z by dividing by 0.6.

The twelve years over which the observations extend are conveniently divided into three four-year periods of which the first and third cover sunspot maxima and the other a minimum. As apparent changes at a time of minimum activity may not actually be significant, attention will be given mainly to differences between the first and third periods. In what follows it is to be understood that these two periods are being differenced when a change is mentioned.

Column 12 indicates that a change of about 25% occurred in the ratio of Washington to Zurich sunspot numbers. Column 9 shows a change of 18 in 10g, and columns 4 and 5 show a change of 37 in s. Hence, the change in sunspot number is due about one-third to changes in group count and two-thirds to changes in spot count. There is internal evidence that the Zurich scale has remained fixed; columns 17 and 18 show a more nearly constant ratio of sunspot number to area for Zurich than for Washington, and 19 and 20 show the same thing for the number of spots per group. To check further Mrs. L. T. Day made new counts of plates previously measured by her in 1938, which confirmed the change in spot count.

It is not as easy to account for the change in group count, however. The Washington value is closely tied in with the photographic group count made at Mount Wilson, g_m, and the change between Mount Wilson and Zurich is almost as large as that between Washington and Zurich. There are no known changes in method at either Mount Wilson or Zurich which readily account for the change in group count.

RELATIONS BETWEEN AREAS AND COUNTS

The data of Table IV lead to some interesting relations between Washington areas and Zurich spot and group counts. Columns 13, 15, 17, and 19 show that the mutual ratios of A_w, g_z, s_z, and r_z to each other are practically constant. The relations are given in Table V in round numbers.

TABLE V

RELATIONS BETWEEN SPOTS, GROUPS, AND AREAS.

(1) $g_z = 0.005 A_w$
(2) $s_z = 0.05 A_w$
(3) $r_z = 0.10 A_w$
(4) $s_z = 10 g_z$.

Since $r = 10g + s$ it is seen that the constant ratio of area to sunspot number found previously follows from the fact that the quantities g and s themselves bear a constant ratio to A. This means that in theory one could form as good an index of solar activity from any linear combination of g and s as from the above. In practice, the choice of $r = 10g + s$ has the advantage of giving about equal weight to spots and groups in forming r. A larger coefficient for g would tend to mask small fluctuations in solar activity while a larger coefficient for s would cause large day-to-day fluctuations. Although the above correlations are based upon one solar cycle, Figure 4 shows that a high degree of correlation between A_w and r_z (or R_z) exists as far back as 1907. Presumably, the other relations have also been the same.

From relation (2) it is found that the area of the average spot counted is 20 millionths of the sun's hemisphere, throughout the cycle. This is equal to 0.4 square degree on the sun's surface. A circle of equivalent area has a diameter of 5500 miles and would have an apparent angular diameter of 12″ if seen at the sun's distance.

DISCUSSION

We have considered two of the many indices of solar activity, namely sunspot numbers and sunspot areas. After averaging out the short period fluctuations it is found that a fixed ratio exists between the two. Both serve about equally well, therefore, to determine the general features of the solar cycle, that is, times of minima and maxima and level of activity. The use of areas in preference to numbers would appear to have some advantage in that they are less affected by short period fluctuations and by long period changes in scale. The short period effects may be eliminated, however, by taking means over a month or more, and the scale can be kept fixed, as at Zurich, if enough controls are used. In many cases, therefore, one index may be used about as well as the other when attempting to correlate solar activity with other phenomena. In some cases either numbers or areas may correlate better, and it may even be necessary to take into account the spot positions. Owing to the inherent observational effects involved caution must be exercised when using solar data over short intervals such as a few days.

NAVY-DPPO PRNC. WASH., D.C.